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G. Bimonte ^(1,2)

and

G. Lozano ⁽¹⁾ ¹

*(1) International Centre for Theoretical Physics, P.O.BOX 586
I-34100 Trieste, ITALY*

(2) INFN, Sezione di Napoli, Napoli, ITALY

Abstract

We consider an Electroweak string in the background of a uniform distribution of cold fermionic matter. As a consequence of the fermion number non-conservation in the Weinberg-Salam model, the string produces a long-range magnetic field.

¹ E-mail addresses: Bimonte@ictp.trieste.it , Lozano@ictp.trieste.it

Since the pionering work by Nielsen and Olesen [1], the study of vortex like solutions in Gauge Theories has attracted a constant interest. This kind of objects is by now believed to play a prominent role in cosmology, for example by providing a mechanism to explain structure formation in the early universe [2].

Until very recently, this type of configurations have been mainly studied in theories with a non trivial vacuum structure where the existence of a topological conservation law ensures its stability. However, as recently stressed by Vachaspati and Barriola [3], the embedding of such configurations in larger gauge groups can be stable even in the absence of such topological conservation laws. This is what happens in the case of the Z-string [4], which results from embedding the U(1) Nielsen Olesen string in the $SU(2) \times U(1)$ theory of Electroweak interactions [5].

In order to understand the role of these configurations during the phase transition it is important to know how they are modified in typical thermodynamical situations such as non-zero temperature and finite matter density. In this letter we shall analyze the case in which the string is placed in a background of cold neutral fermionic matter, a situation in which, due to the V-A character of the Electroweak interactions, the fermion sector of the model manifests in a non trivial way.

To illustrate the kind of phenomena that occur, let us begin with the simpler $U(1)$ model and study the way in which a constant fermionic density affects the Nielsen Olesen string. We consider the system described by the Lagrangian density,

$$L = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + (D_\mu\phi)(D^\mu\phi) - \lambda(\phi^2 - \eta^2)^2 + L_f . \quad (1)$$

Here the covariant derivative for the scalar complex field ϕ is defined as

$$D_\mu\phi = (\partial_\mu + iqZ_\mu)\phi \quad (2)$$

and Z_μ and $Z_{\mu\nu}$ are the vector potential and field strength.

The interaction of the fermions with the gauge fields is governed by the Lagrangian density

$$L_f = \bar{\psi}_L^- i\sigma_\mu(\partial_\mu - i\frac{q}{2}Z_\mu)\psi_L^- + \bar{\psi}_L^+ i\sigma_\mu(\partial_\mu + i\frac{q}{2}Z_\mu)\psi_L^+ \quad (3)$$

where ψ_L^- and ψ_L^+ are left handed fermions with opposite charge and σ_μ are the Pauli matrices. Following

Rubakov and Tavkhelidze [6], we will consider the case in which there is a cold (zero temperature) and neutral ($n^+ = n^- = n/2$) background of fermions.

In the weakly coupled regime, the bosonic field can be treated classically and its contribution to the energy (in the static case) is simply given by

$$E_b = \int d^3x \left[\frac{1}{4} Z_{ij}^2 + |D_i \phi|^2 + \lambda(\phi^2 - \eta^2)^2 \right] . \quad (4)$$

As shown in Refs. [6], [7] the presence of a finite density manifests through the appearance in the effective energy of a Chern Simons term [8] which results from the integration of the fermionic determinant

$$E_f = -\frac{k}{2} \int d^3x \epsilon^{ijk} Z_{ij} Z_k \quad (5)$$

the coefficient k being related to the chemical potential μ and fermion density n by

$$k = \frac{\mu q^2}{16\pi^2} = \frac{q^2}{16\pi^2} (3\pi^2 n)^{\frac{1}{3}} . \quad (6)$$

The effective energy

$$E = E_b + E_f \quad (7)$$

was originally used in Refs [6] [9] where it was shown that the trivial vacuum becomes unstable for chemical potentials larger than the critical value

$$\mu_{crit} = \frac{32\pi^2}{q^2} m_v \quad (8)$$

where the mass of the gauge particle is $m_v^2 = 2q^2\eta^2$. We will instead consider the case of small chemical potentials $\mu \leq \mu_{crit}$ in order to study how the fermionic background modifies the Nielsen Olesen string.

Notice that by making in (7) the transformation

$$A_3 \rightarrow iA_0 \quad x^3 \rightarrow -ix^0 \quad k \rightarrow ik \quad (9)$$

we obtain the *action* of the 2+1 dimensional Chern-Simons-Higgs system. Vortex like solutions in this and related models have been object of much study in recent years mainly in the context of High Temperature Superconductivity and the Quantum Hall Effect [10] [11]. A general feature of these solutions is that as

a result of the Chern Simons interaction the vortex acquires an electric charge and consequently a radial electric field.

The transformation (9) corresponds to the combined effect of an Euclidean rotation and a duality transformation in the $x_0 - x_3$ plane. As a consequence, what in the $2 + 1$ dimensional case corresponded to an electric charge will now become a flux of current along the string and the radial electric field will be now converted into a tangential magnetic field. Making the ansatz

$$\phi = f(r) \exp(im\theta) \quad \mathbf{Z} = Z_\theta \mathbf{e}_\theta + Z_3 \mathbf{e}_3 \quad (10)$$

the equations of motion then read

$$(rZ_3')' - 2q^2 r f^2 Z_3 + 2kZ_\theta' = 0 \quad (11)$$

$$\left(\frac{Z_\theta'}{r}\right)' - 2q \frac{m + qZ_\theta}{r} f^2 - 2kZ_3' = 0 \quad (12)$$

$$\frac{1}{r}(rf')' - \frac{(m + qZ_\theta)^2}{r^2} f - q^2 Z_3^2 f - 2\lambda f(f^2 - \eta^2) = 0 \quad (13)$$

subject to the boundary conditions

$$Z_3(\infty) = 0 \quad Z_\theta(\infty) = -\frac{m}{q} \quad f(\infty) = \eta \quad (14)$$

$$Z_3(0) < \infty \quad Z_\theta(0) = 0 \quad f(0) = 0 \quad . \quad (15)$$

Although an exact solution to the equations cannot be obtained analytically, it is possible (in complete analogy to the $2 + 1$ dimensional case [10] [11]) to express the first terms of an expansion in k in a closed form involving the solutions of the Nielsen Olesen vortex

$$Z_\theta = Z_\theta^{NO} - \frac{k^2 r^2}{2} (Z_\theta^{NO} + \frac{m}{q}) + O(k^4) \quad (16)$$

$$Z_3 = -k(Z_\theta^{NO} + \frac{m}{q}) + O(k^3) \quad (17)$$

$$f = f^{NO} + O(k^4) \quad (18)$$

where the superscript NO refers to the Nielsen Olesen solutions, that is, solutions to equations (11-13) with $k = 0$. Thus, we see that the leading effect is the appearance of a tangential component of the *quasi magnetic* field C_i (we reserve the name magnetic field for later when we introduce the unbroken $U(1)$ of

electromagnetism)

$$C_i = \frac{1}{2}\epsilon_{ijk}Z_{jk} \quad (19)$$

$$C_\theta = -rZ'_3 \quad (20)$$

which can be associated to a flux of current flowing along the string

$$J = \int d^2x j_3 \quad j_l = iq(\phi D_l \phi^* - \phi^* D_l \phi) \quad (21)$$

Using the boundary conditions, it can be easily shown that

$$J = 2k\Phi \quad (22)$$

where Φ is the flux of the quasi-magnetic field

$$\Phi = \int d^2x C_3 = -2\pi \frac{m}{q} \quad (23)$$

The asymptotic behavior for the gauge fields at infinity can be derived from (11-13)

$$Z_3 \rightarrow \frac{ic}{\sqrt{r}}e^{-\mu r} + h.c. \quad (24)$$

$$Z_\theta \rightarrow -\frac{m}{q} + (c\sqrt{r}e^{-\mu r} + h.c.) \quad (25)$$

where

$$\mu = \sqrt{(m_v^2 - k^2)} - ik \quad m_v^2 = 2q^2\eta^2 \quad (26)$$

Comparing this result with the 2 + 1 dimensional case, we see that as a consequence of the replacement $k \rightarrow ik$, μ has become a complex number. Notice that as far as $k^2 \leq m_v^2$ or equivalently $\mu \leq \mu_{crit}$, μ has a positive real part. In addition, this real part is smaller compared to the case $k = 0$ which clearly corresponds to the anti-screening effect of the Chern Simons term. We then see, that the value $\mu = \mu_{crit}$ corresponds to the point for which this effect completely overwhelms the screening originating from the Higgs mechanism.

As for the Higgs field, its asymptotic is

$$f \rightarrow \frac{D_1}{r}e^{-\mu r} + \frac{D_2}{\sqrt{r}}e^{-m_H r} + h.c. \quad (27)$$

$$m_H^2 = 4\lambda\eta^2 \quad (28)$$

and depending on the ratio $\frac{|\mu|}{m_H}$ the behavior at infinity will be determined by the first or second term [15].

We can now turn to the study of the Electroweak case. The contribution of the bosons to the static effective energy is

$$E_b = \int d^3x \left[\frac{1}{4} (W_{ij}^a)^2 + \frac{1}{4} B_{ij}^2 + |D_i \Phi|^2 + \lambda (\Phi^2 - \eta^2)^2 \right] \quad (29)$$

where the covariant derivative of the Higgs doublet and the strengths of the $SU(2) \times U(1)$ gauge fields are defined as

$$D_i \Phi = (\partial_i - i \frac{g}{2} \tau^a W_i^a - i \frac{g'}{2} B_i) \Phi \quad (30)$$

$$W_{ij}^a = \partial_i W_j^a - \partial_j W_i^a + g \epsilon^{abc} W_i^a W_j^b \quad (31)$$

$$B_{ij} = \partial_i B_j - \partial_j B_i \quad (32)$$

Assuming a cold background of fermions satisfying

$$n_{e_L}^{(i)} = n_{\nu}^{(i)} = n_{d_L}^{(i)a} = n_{u_L}^{(i)a} = n_{u_R}^{(i)a} = n_{d_R}^{(i)a} = n_{e_R}^{(i)} \quad (33)$$

(where $i = 1, \dots, f$ and $a = 1, \dots, 3$ denote the family and color indices respectively) their contribution to the effective energy is given by [6], [7]:

$$E_F = -4f\mu [N_{CS}(W) - N_{CS}(B)] \quad (34)$$

$$N_{CS}(W) = \frac{g^2}{32\pi^2} \int d^3x \epsilon^{ijk} (W_{ij}^a W_k^a - \frac{g}{3} \epsilon^{abc} W_i^a W_j^b W_k^c) \quad (35)$$

$$N_{CS}(B) = \frac{g'^2}{32\pi^2} \int d^3x \epsilon^{ijk} B_{ij} B_k \quad (36)$$

The effects of the Chern Simons term on the ground state of the Weinberg Salam model in an external magnetic field and its influence on the phenomenon of W-condensation [14] has been recently analyzed by Poppitz [15]. We will instead concentrate here on the influence of this term on the Z-string configuration and consequently we will set to zero the charged W bosons and the upper component of the Higgs field. Denoting as ϕ the lower component of the Higgs, the energy takes the form

$$E = \int d^3x \left[\frac{1}{4} (F_{ij})^2 + \frac{1}{4} Z_{ij}^2 + |D_i \phi|^2 + \lambda (\phi^2 - \eta^2)^2 - \epsilon^{ijk} (k_1 Z_i F_{jk} + \frac{k_2}{2} Z_i Z_{jk}) \right] \quad (37)$$

where we have introduced the electromagnetic vector potential A_i and the neutral vector potential Z_i by

$$A_i = W_i^3 \sin \theta + B_i \cos \theta \quad (38)$$

$$Z_i = W_i^3 \cos \theta - B_i \sin \theta \quad (39)$$

$$\tan \theta = \frac{g'}{g} \quad (40)$$

and k_1 and k_2 are defined as

$$k_1 = \frac{gg'}{4\pi^2} \mu f \quad k_2 = \frac{g^2 - g'^2}{4\pi^2} \mu f \quad . \quad (41)$$

Notice the absence of Chern Simons term associated to the field A_i , a fact which can be traced back to the vector character of the electromagnetic interactions. Due to the presence of the cross Chern Simons term, the field Z_i will now act as a source for A_i and it is not any more consistent (as in the purely bosonic case [4]) to set it to zero. Indeed, the equations for A_i are

$$\partial_i F^{il} = -k_1 \epsilon^{kjl} Z_{kj} \quad . \quad (42)$$

For μ (i.e. k) small compared with the particle masses, we can neglect the back reaction of the condensate on the string. Then, replacing the right hand side of (42) by its value in the Nielsen Olesen case, we obtain a tangential magnetic field

$$\mathbf{H} = 2k_1 \frac{Z_\theta^{NO}}{r} \mathbf{e}_\theta \quad . \quad (43)$$

At, large distances, where Z_θ^{NO} goes to $-m/q$ ($q = \frac{\sqrt{g^2 + g'^2}}{2}$)

$$\mathbf{H} = -\frac{em\mu f}{\pi^2 r} \mathbf{e}_\theta \quad (44)$$

which coincides with the magnetic field generated by an infinitely long wire carrying a current

$$I = -\frac{em\mu f}{2\pi^2} \quad . \quad (45)$$

As in the abelian case, there will be a tangential component of the quasi-magnetic field

$$\mathbf{C} = -r Z'_3 \mathbf{e}_\theta = -k_2 r (Z_\theta^{NO})' \mathbf{e}_\theta \quad . \quad (46)$$

We can estimate the maximum value of the magnetic field to be

$$H_{max} \sim \frac{k_1 m_v m}{q} \quad . \quad (47)$$

As we mentioned before, all these considerations are valid for small chemical potentials and strictly speaking for small distances from the core of the string.

A more complete analysis would follow from the numerical study of the equations of motion which result from the ansatz

$$\phi = f(r) \exp(im\theta) \quad \mathbf{Z} = Z_\theta \mathbf{e}_\theta + Z_3 \mathbf{e}_3 \quad \mathbf{A} = A_\theta(r) \mathbf{e}_\theta + A_3(r) \mathbf{e}_3 \quad , \quad (48)$$

$$(rZ'_3)' - 2q^2 r f^2 Z_3 + 2k_2 Z'_\theta + 4k_1^2 r Z_3 = 0 \quad (49)$$

$$\left(\frac{Z'_\theta}{r}\right)' - 2q \frac{m + qZ_\theta}{r} f^2 - 2k_2 Z'_3 + 4k_1^2 \frac{Z_\theta}{r} = 0 \quad (50)$$

$$\frac{1}{r}(rf')' - \frac{(m + qZ_\theta)^2}{r^2} f - q^2 Z_3^2 f - 2\lambda f(f^2 - \eta^2) = 0 \quad . \quad (51)$$

Once the solutions are known, the electromagnetic potentials are calculated by direct integration (see eqs (42)). At infinity, due to the long range character of electromagnetism, we expect a substantial modification of the Z-string profile. Indeed, the asymptotic behavior of the massive fields will turn from exponential to power like and additionally there will be a modification of the total Z-flux. To the leading order in the chemical potential we obtain

$$Z_\theta \rightarrow -\frac{m}{q} - k_1^2 c_1 \quad (52)$$

$$Z_3 \rightarrow -\frac{c_3 k_1^6 k_2}{r^4} \quad (53)$$

$$f \rightarrow \eta - \frac{c_4 k_1^4}{r^2} \quad (54)$$

$$A_3 \rightarrow \frac{2m}{q} k_1 \ln(r) \quad (55)$$

$$A_\theta \rightarrow -k_1^7 k_2 c_3 \frac{1}{r^2} \quad . \quad (56)$$

Summarizing, we have shown that the presence of cold neutral matter considerably modifies the original Z-string profile leading to the appearance of long range magnetic fields associated with currents flowing along the string.

Clearly, an important issue to be addressed concerns the stability of these configurations. As it was established in Ref. [13], the bare Electroweak string is unstable for realistic values of the coupling

constants. The question is then to study whether the fermions can improve the stability of the Z-string. Although the answer of this issue in the full Electroweak model seems rather complicated, an encouraging result comes from the analysis of the semilocal string. In fact, for this case, stability is improved. This can be shown by looking at the potential term in the Schroedinger-like equation satisfied by the perturbations and noticing that it is less negative than in the bare case. It would be interesting to explore in the Electroweak theory the region of stability as a function of the coupling constants and the fermion density (and eventually include temperature corrections) in order to establish the role of these configurations in a cosmological setting.

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